

تمرین های زیر را حل کنید:

12, 15, 20, 24, 28, 55, 58, 47, 52, 104, 126, 131, 144, 146, 155

# **PROBLEMS**

**4.1** and **4.2** Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

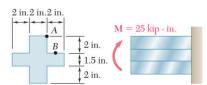


Fig. P4.1

- **4.3** Using an allowable stress of 16 ksi, determine the largest couple that can be applied to each pipe.
- **4.4** A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple  $\mathbf{M}_z$  that can be applied to the bar.

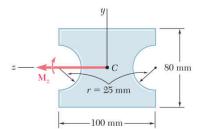


Fig. P4.4

**4.5** A beam of the cross section shown is extruded from an aluminum alloy for which  $\sigma_Y = 250$  MPa and  $\sigma_U = 450$  MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.

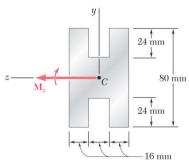
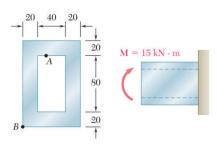


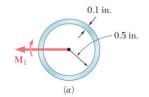
Fig. P4.5

**4.6** Solve Prob. 4.5, assuming that the beam is bent about the y axis.



Dimensions in mm

Fig. P4.2



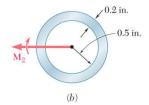


Fig. P4.3

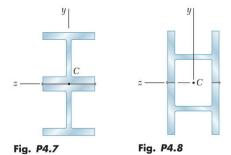




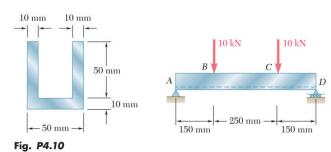
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# **4.7 and 4.8** Two W4 $\times$ 13 rolled sections are welded together as shown. Knowing that for the steel alloy used, $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple

that can be applied when the assembly is bent about the z axis.



**4.9 through 4.11** Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



**4.12** Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is  $6~\mathrm{kN}$  · m, determine the total force acting on the top flange.

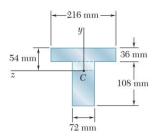
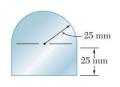


Fig. P4.12 and P4.13

**4.13** Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN·m, determine the total force acting on the shaded portion of the web.



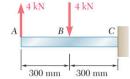


Fig. P4.9

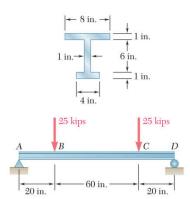


Fig. P4.11



نام درس: مقاومت مصالح۱

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**4.14** Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 50 kip  $\cdot$  in., determine the total force acting (a) on the top flange (b) on the shaded portion of the web.

**4.15** The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

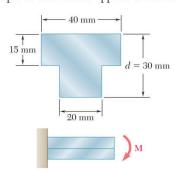
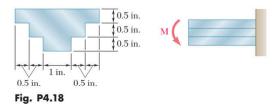


Fig. P4.15

- **4.16** Solve Prob. 4.15, assuming that d = 40 mm.
- **4.17** Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.
- 4.18 Knowing that for the casting shown the allowable stress is 5 ksi in tension and 18 ksi in compression, determine the largest couple M that can be applied.



**4.19 and 4.20** Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

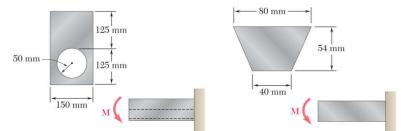


Fig. P4.19 Fig. P4.20

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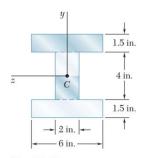


Fig. P4.14

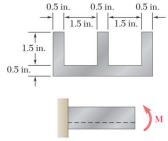


Fig. P4.17

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Fig. P4.21

- **4.21** A steel band saw blade, that was originally straight, passes over 8-in.-diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.018 in. thick and 0.625 in. wide. Use  $E=29\times10^6$  psi.
- **4.22** Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a rod, when the rod, which is initially straight, is wrapped on a spool, (b) the corresponding bending moment in the rod. Use  $E=29\times10^6$  psi.

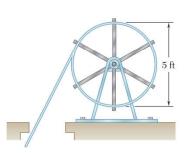


Fig. P4.22

 $\begin{array}{c|c}
M' & M \\
\hline
r & & \\
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 & & \\
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Fig. P4.23

- **4.23** A 900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine (a) the maximum thickness t of the strip if the allowable stress of the steel is 420 MPa, (b) the corresponding moment M of the couples. Use E=200 GPa.
- **4.24** A 60-N·m couple is applied to the steel bar shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the bar. (b) Solve part a, assuming that the couple is applied about the y axis. Use E=200 GPa.

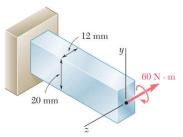


Fig. P4.24

**4.25** A couple of magnitude *M* is applied to a square bar of side *a*. For each of the orientations shown, determine the maximum stress and the curvature of the bar.

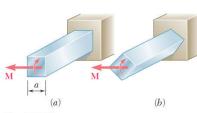


Fig. P4.25

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**4.26** A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where  $h=0.9h_0$ , express the maximum stress in the bar in the form  $\sigma_m=k\sigma_0$  where  $\sigma_0$  is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

- **4.27** In Prob. 4.26, determine (a) the value of h for which the maximum stress  $\sigma_m$  is as small as possible, (b) the corresponding value of k.
- **4.28** A couple **M** will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b for which (a) the maximum stress  $\sigma_m$  will be as small as possible, (b) the radius of curvature of the beam will be
- **4.29** For the aluminum bar and loading of Sample Prob. 4.1, determine (a) the radius of curvature  $\rho'$  of a transverse cross section, (b) the angle between the sides of the bar that were originally vertical. Use  $E=10.6\times10^6$  psi and  $\nu=0.33$ .
- **4.30** For the bar and loading of Example 4.01, determine (a) the radius of curvature  $\rho$ , (b) the radius of curvature  $\rho'$  of a transverse cross section, (c) the angle between the sides of the bar that were originally vertical. Use  $E=29\times10^6$  psi and  $\nu=0.29$ .
- **4.31** A W200  $\times$  31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN  $\cdot$  m. Knowing that E=200 GPa and  $\nu=0.29$ , determine (a) the radius of curvature  $\rho$ , (b) the radius of curvature  $\rho'$  of a transverse cross section.

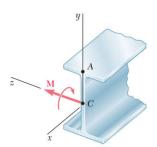


Fig. P4.31

**4.32** It was assumed in Sec. 4.3 that the normal stresses  $\sigma_y$  in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (a) derive an approximate expression for  $\sigma_y$  as a function of y, (b) show that  $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$  and, thus, that  $\sigma_y$  can be neglected in all practical situations. (*Hint*: Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume that the distribution of the stress  $\sigma_x$  is still linear.)

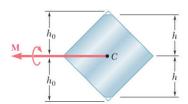


Fig. P4.26

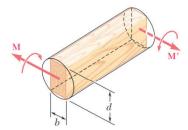


Fig. P4.28

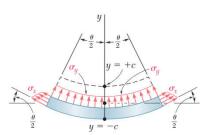


Fig. P4.32

# **PROBLEMS**

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

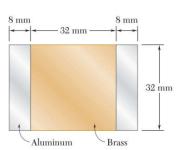


Fig. P4.33

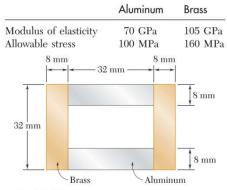


Fig. P4.34

- 4.35 and 4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.
  - 4.35 Bar of Prob. 4.33.
  - **4.36** Bar of Prob. 4.34.
- **4.37** and **4.38** Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity Allowable stress	$2 \times 10^6  \mathrm{psi}$ 2000 psi	$29 \times 10^6 \text{ psi}$ 22 ksi
3 in. 3 in. ½ in.	$5 \times \frac{1}{2}$ $5 \times \frac{1}{2}$ $6 \text{ in}$ Fig. P4	10 in.

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4.39 and 4.40 A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment *M* = 1500 N ⋅ m, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

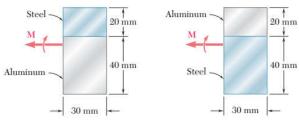


Fig. P4.39

Fig. P4.40

**4.41** and **4.42** The  $6 \times 12$ -in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is  $1.8 \times 10^6$  psi and for steel is  $29 \times 10^6$  psi. Knowing that the beam is bent about a horizontal axis by a couple of moment M = 450 kip  $\cdot$  in., determine the maximum stress in (a) the wood, (b) the steel.

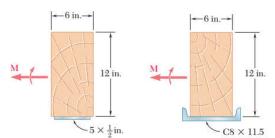


Fig. P4.41

Fig. P4.42

- **4.43 and 4.44** For the composite beam indicated, determine the radius of curvature caused by the couple of moment 1500 N  $\cdot$  m.
  - **4.43** Beam of Prob. 4.39.
  - **4.44** Beam of Prob. 4.40.
- 4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.
  - 4.45 Beam of Prob. 4.41.
  - **4.46** Beam of Prob. 4.42.
- **4.47** The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.
- **4.48** Solve Prob. 4.47, assuming that the 300-mm width is increased to 350 mm.

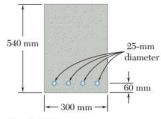


Fig. P4.47

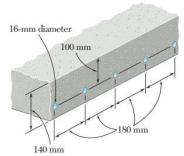


Fig. P4.49

- 4.49 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9 MPa for the concrete and 120 MPa for the steel, determine the largest bending moment in a portion of slab
- 4.50 Solve Prob. 4.49, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.
- A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is  $3 \times 10^6$  psi for the concrete and  $29 \times$ 106 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

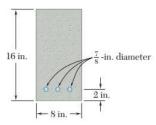


Fig. P4.51

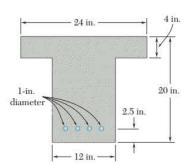


Fig. P4.52

- 4.52 Knowing that the bending moment in the reinforced concrete beam is  $\pm 100 \text{ kip} \cdot \text{ft}$  and that the modulus of elasticity is  $3.625 \times 10^6 \text{ psi}$ for the concrete and 29  $\times$  10<sup>6</sup> psi for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.
- 4.53 The design of a reinforced concrete beam is said to be balanced if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses  $\sigma_s$  and  $\sigma_c$ . Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \frac{\sigma_s E_c}{\sigma_c E_s}}$$

where  $E_c$  and  $E_s$  are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

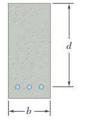


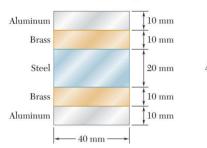
Fig. P4.53 and P4.54

4.54 For the concrete beam shown, the modulus of elasticity is 3.5 ×  $10^6$  psi for the concrete and  $29 \times 10^6$  psi for the steel. Knowing that b = 8 in. and d = 22 in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (a) the required area A, of the steel reinforcement if the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.53 for definition of a balanced beam.)

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**4.55** and **4.56** Five metal strips, each 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.



Brass 10 mmSteel 10 mmAluminum 20 mmSteel 10 mmBrass 10 mm

Fig. P4.55

Fig. P4.56

**4.57** The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is  $15 \times 10^6$  psi for the brass and  $10 \times 10^6$  psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip  $\cdot$  in., determine the maximum stress (a) in the brass, (b) in the aluminum.

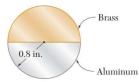


Fig. P4.57

- **4.58** A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by a couple of moment 500 N·m, determine the maximum stress (a) in the aluminum, (b) in the steel.
- **4.59** The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment  $M=600~\mathrm{N}\cdot\mathrm{m}$ , determine the maximum (a) tensile stress, (b) compressive stress.
- \*4.60 A rectangular beam is made of material for which the modulus of elasticity is  $E_t$  in tension and  $E_c$  in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

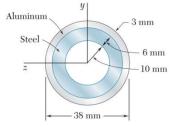


Fig. P4.58

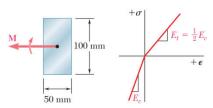


Fig. P4.59

# **PROBLEMS**

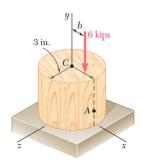


Fig. P4.99

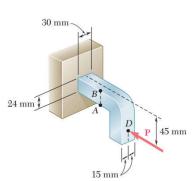


Fig. P4.101

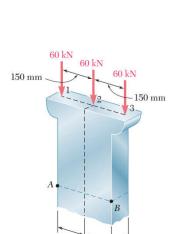


Fig. P4.104

120 mm

120 mm

- **4.99** A short wooden post supports a 6-kip axial load as shown. Determine the stress at point A when (a) b = 0, (b) b = 1.5 in., (c) b = 3 in.
- **4.100** As many as three axial loads each of magnitude P=10 kips can be applied to the end of a W8  $\times$  21 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

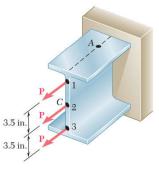


Fig. P4.100

- **4.101** Knowing that the magnitude of the horizontal force  $\mathbf{P}$  is 8 kN, determine the stress at (a) point A, (b) point B.
- **4.102** The vertical portion of the press shown consists of a rectangular tube of wall thickness t=10 mm. Knowing that the press has been tightened on wooden planks being glued together until P=20 kN, determine the stress at (a) point A, (b) point B.

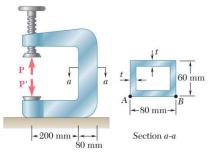


Fig. P4.102

- **4.103** Solve Prob. 4.102, assuming that t = 8 mm.
- **4.104** Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 1 and 2 only.

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**4.105** Knowing that the allowable stress in section *ABD* is 10 ksi, determine the largest force **P** that can be applied to the bracket shown.

**4.106** Portions of a  $\frac{1}{2} \times \frac{1}{2}$ -in. square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 15 ksi, determine the maximum load that can be applied to each component.

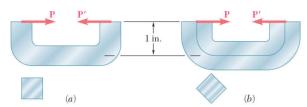


Fig. P4.106

**4.107** The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that P = 100 kN and a = 40 mm, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

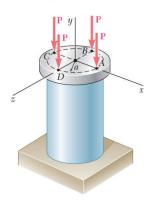


Fig. P4.107

- **4.108** A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that a=30 mm, d=20 mm, and  $\sigma_{\rm all}=60$  MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.
- **4.109** A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude P=18 kN are applied at the centers of the ends of the bar. Knowing that a=30 mm and  $\sigma_{\rm all}=135$  MPa, determine the smallest allowable depth d of the milled portion of the bar.
- **4.110** A short column is made by nailing two 1 × 4-in. planks to a 2 × 4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (a) the column is as described, (b) plank 1 is removed, (c) both planks are removed.

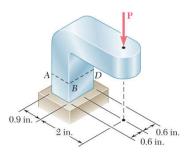


Fig. P4.105

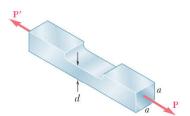


Fig. P4.108 and P4.109

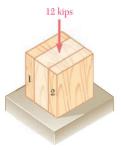


Fig. P4.110



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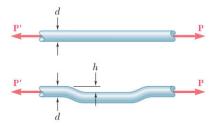


Fig. P4.111 and P4.112

- 4.111 An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.
- **4.112** An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the tube when it is straight, determine the largest offset that can be used.
- **4.113** A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (a) the largest force **P** that can be applied to the element, (b) the corresponding location of the neutral axis. Given: The centroid of the cross section is at C and  $I_z = 4195 \text{ mm}^4$ .

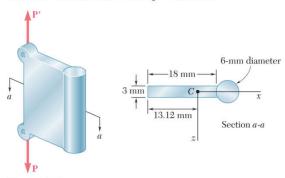


Fig. P4.113

**4.114** A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are  $\sigma_{\rm all} = +5$  ksi and  $\sigma_{\rm all} = -12$  ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

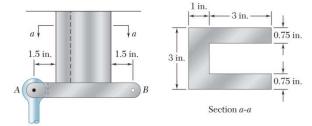


Fig. P4.114

- **4.115** Solve Prob. 4.114, assuming that the vertical rod is attached at point *B* instead of point *A*.
- 4.116 Three steel plates, each of 25 × 150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section and that the allowable stress is 100 MPa, determine the largest force P (a) that could be applied to the original column, (b) that can be applied to the modified column.

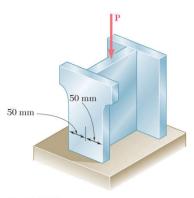


Fig. P4.116

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**4.117** A vertical force **P** of magnitude 20 kips is applied at point C located on the axis of symmetry of the cross section of a short column. Knowing that y=5 in., determine (a) the stress at point A, (b) the stress at point B, (c) the location of the neutral axis.

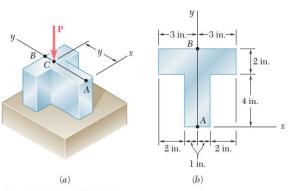


Fig. P4.117 and P4.118

- **4.118** A vertical force  $\mathbf{P}$  is applied at point C located on the axis of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.
- **4.119** Knowing that the clamp shown has been tightened until P = 400 N, determine (a) the stress at point A, (b) the stress at point B, (c) the location of the neutral axis of section a-a.

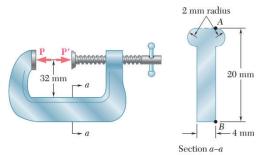


Fig. P4.119

**4.120** The four bars shown have the same cross-sectional area. For the given loadings, show that (a) the maximum compressive stresses are in the ratio 4:5:7:9, (b) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note*: the cross section of the triangular bar is an equilateral triangle.)

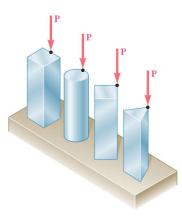


Fig. P4.120

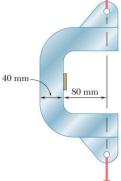


Fig. P4.121

- **4.121** The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that the strain on the inner edge was measured and found to be 450  $\mu$ , determine the magnitude P of the forces. Use E = 200 GPa.
- **4.122** An eccentric force **P** is applied as shown to a steel bar of  $25 \times 90$ -mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350 \ \mu$$
  $\epsilon_B = -70 \ \mu$ 

Knowing that E = 200 GPa, determine (a) the distance d, (b) the magnitude of the force P.

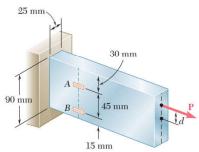


Fig. P4.122

4.123 Solve Prob. 4.122, assuming that the measured strains are

$$\epsilon_A = +600 \ \mu$$
  $\epsilon_B = +420 \ \mu$ 

**4.124** A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads P and Q are applied as shown. The strains at two points A and B on the centerline of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -550 \times 10^{-6}$$
 in./in.  $\epsilon_B = -680 \times 10^{-6}$  in./in.

Knowing that  $E = 29 \times 10^6$  psi, determine the magnitude of each

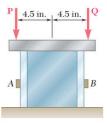


Fig. P4.124

4.125 Solve Prob. 4.124, assuming that the measured strains are

$$\epsilon_A = +35 \times 10^{-6}$$
 in./in. and  $\epsilon_B = -450 \times 10^{-6}$  in./in.

4.126 The eccentric axial force P acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For P = 60 kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A.

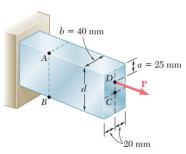


Fig. P4.126



# **PROBLEMS**

**4.127 through 4.134** The couple **M** is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

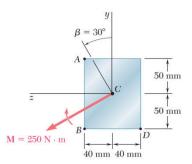


Fig. P4.127

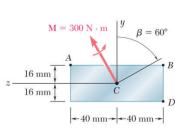


Fig. P4.128

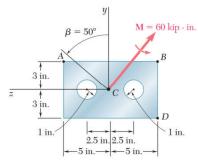


Fig. P4.129

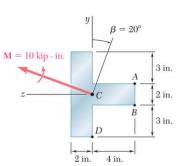


Fig. P4.130

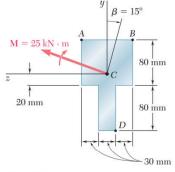


Fig. P4.131

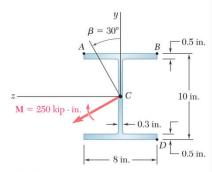


Fig. P4.132

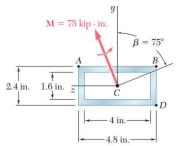


Fig. P4.133

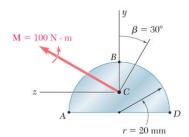


Fig. P4.134





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**4.135 through 4.140** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

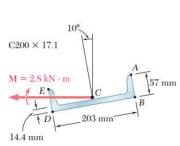


Fig. P4.135

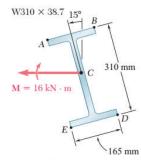


Fig. P4.136

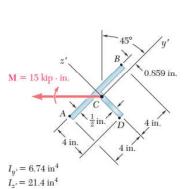
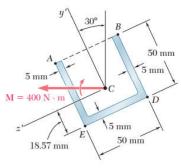


Fig. P4.137



 $I_{y'} = 281 \times 10^3 \text{ mm}^4$  $I_{z'} = 176.9 \times 10^3 \text{ mm}^4$ 

Fig. P4.138

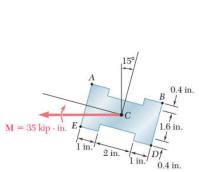


Fig. P4.139

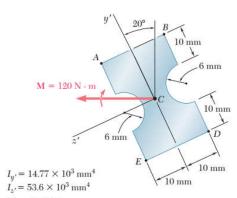


Fig. P4.140



\*4.141 through \*4.143 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

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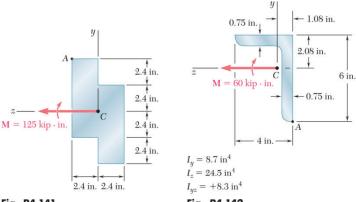
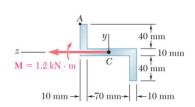




Fig. P4.142



 $I_y = 1.894 \times 10^6 \; \mathrm{mm}^4$  $I_z = 0.614 \times 10^6 \,\mathrm{mm}^4$  $I_{yz} = +0.800 \times 10^6 \, \mathrm{mm}^4$ 

Fig. P4.143

- 4.144 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.
- **4.145** Solve Prob. 4.144, assuming that the 28-kN force at point E is removed.
- **4.146** A rigid circular plate of 125-mm radius is attached to a solid 150  $\times$ 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with  $\theta =$ 30°, determine (a) the stress at point A, (b) the stress at point B, (c) the point where the neutral axis intersects line ABD.

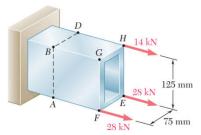


Fig. P4.144

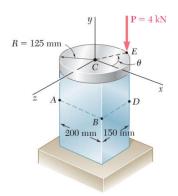


Fig. P4.146

**4.147** In Prob. 4.146, determine (a) the value of  $\theta$  for which the stress at D reaches its largest value, (b) the corresponding values of the stress at A, B, C, and D.

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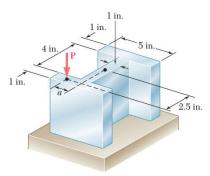


Fig. P4.148 and P4.149

 $\begin{array}{c|c}
y & 0.5 \text{ in.} \\
\hline
z & M_0 & C & 1.43 \text{ in.} \\
\hline
0.5 \text{ in.} & 5 \text{ in.}
\end{array}$ 

5 in

Fig. P4.152

1.43 in.-

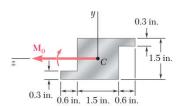


Fig. P4.154

- **4.148** Knowing that P = 90 kips, determine the largest distance a for which the maximum compressive stress does not exceed 18 ksi.
- **4.149** Knowing that a = 1.25 in., determine the largest value of **P** that can be applied without exceeding either of the following allowable stresses:

$$\sigma_{\mathrm{ten}} = 10$$
 ksi  $\sigma_{\mathrm{comp}} = 18$  ksi

**4.150** The Z section shown is subjected to a couple  $\mathbf{M}_0$  acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 80 MPa. Given:  $I_{\text{max}} = 2.28 \times 10^{-6} \text{ m}^4$ ,  $I_{\text{min}} = 0.23 \times 10^{-6} \text{ m}^4$ , principal axes  $25.7^{\circ}$  and  $64.3^{\circ}$   $\angle$ .

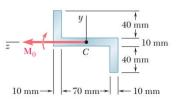


Fig. P4.150

- **4.151** Solve Prob. 4.150, assuming that the couple  $\mathbf{M}_0$  acts in a horizontal plane.
- **4.152** A beam having the cross section shown is subjected to a couple  $\mathbf{M}_0$  that acts in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress in the beam is not to exceed 12 ksi. Given:  $I_y = I_z = 11.3$  in<sup>4</sup>, A = 4.75 in<sup>2</sup>,  $k_{\min} = 0.983$  in. (Hint: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations  $I_{\min} = Ak^2_{\min}$  and  $I_{\min} + I_{\max} = I_y + I_z$ .)
- **4.153** Solve Prob. 4.152, assuming that the couple  $\mathbf{M}_0$  acts in a horizontal plane.
- **4.154** An extruded aluminum member having the cross section shown is subjected to a couple acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 12 ksi. Given:  $I_{\text{max}} = 0.957 \text{ in}^4$ ,  $I_{\text{min}} = 0.427 \text{ in}^4$ , principal axes  $29.4^{\circ} \angle 100.60 = 100.60$
- **4.155** A couple  $\mathbf{M}_0$  acting in a vertical plane is applied to a W12  $\times$  16 rolled-steel beam, whose web forms an angle  $\theta$  with the vertical. Denoting by  $\sigma_0$  the maximum stress in the beam when  $\theta=0$ , determine the angle of inclination  $\theta$  of the beam for which the maximum stress is  $2\sigma_0$ .

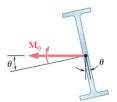


Fig. P4.155

**4.156** Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of a rectangular cross section, the neutral axis will lie along the other diagonal.

**4.157** A beam of unsymmetric cross section is subjected to a couple  $\mathbf{M}_0$  acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_yI_z - I_{yz}^2} M_y$$

where  $I_y$ ,  $I_z$ , and  $I_{yz}$  denote the moments and product of inertia of the cross section with respect to the coordinate axes, and  $M_y$  the moment of the couple.

**4.158** A beam of unsymmetric cross section is subjected to a couple  $\mathbf{M}_0$  acting in the vertical plane xy. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_yI_z - I_{yz}^2}M_z$$

where  $I_y$ ,  $I_z$ , and  $I_{yz}$  denote the moments and product of inertia of the cross section with respect to the coordinate axes, and  $M_z$  the moment of the couple.

**4.159** (a) Show that, if a vertical force  $\mathbf{P}$  is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$

where  $r_z$  and  $r_x$  denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further show that, if a vertical force  $\mathbf{Q}$  is applied at any point located on line BD, the stress at point A will be zero.

**4.160** (a) Show that the stress at corner A of the prismatic member shown in Fig. P4.160a will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

(b) Further show that, if no tensile stress is to occur in the member, the force  ${\bf P}$  must be applied at a point located within the area bounded by the line found in part a and three similar lines corresponding to the condition of zero stress at B, C, and D, respectively. This area, shown in Fig. P4.160b, is known as the kern of the cross section.

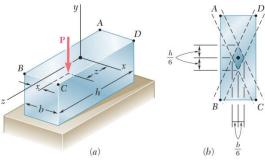


Fig. P4.160

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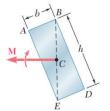


Fig. P4.156

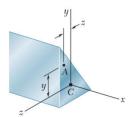


Fig. P4.157 and P4.158

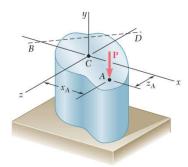


Fig. P4.159