

دانشگاه سواد

مقاومت مصالح

مبحث پنجم:

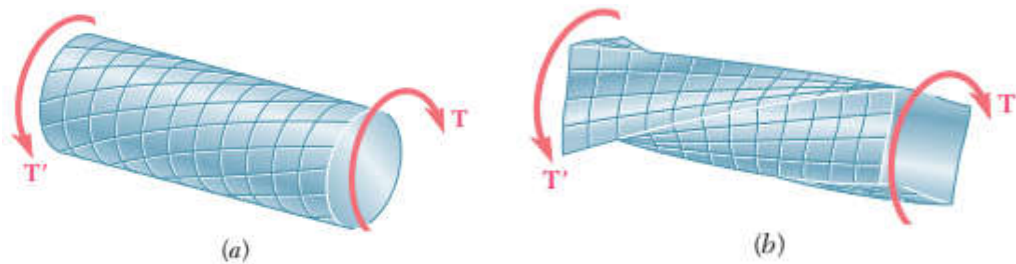
پیش

مدرس: دکتر محمد رضا میرجلیلی

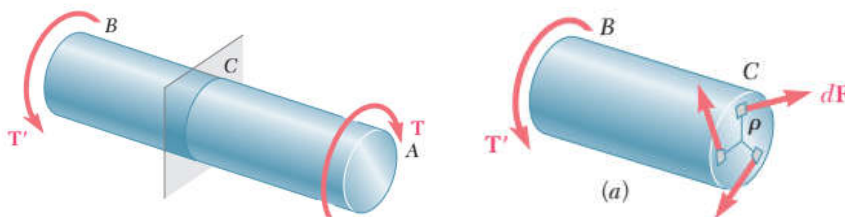


۵-۱- پیچش در شفت های با مقطع دایره

مقاطع دایره ای وقتی تحت پیچش قرار می گیرند همان دایره باقی می ماند ولی مقاطع مستطیلی یا مربعی تحت پیچش تغییر شکل می دهند و اصطلاحاً دچار اعوجاج می شوند.



در این بخش ابتدا مقاطع دایره ای بررسی می شوند.



$$\int \rho dF = T$$

$$\int \rho (\tau dA) = T$$

$$\gamma = \frac{\rho \phi}{L}$$

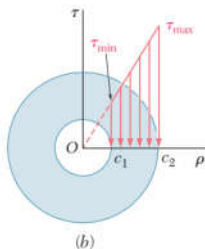
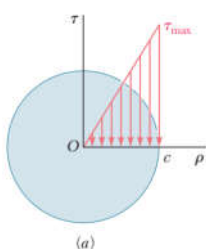
$$\gamma_{\max} = \frac{c \phi}{L}$$

$$\gamma = \frac{\rho}{c} \gamma_{\max}$$

توزیع کرنش در مقطع دایره نسبت به شعاع خطی است.

$$\tau = G \gamma$$

$$\tau = \frac{\rho}{c} \tau_{\max}$$

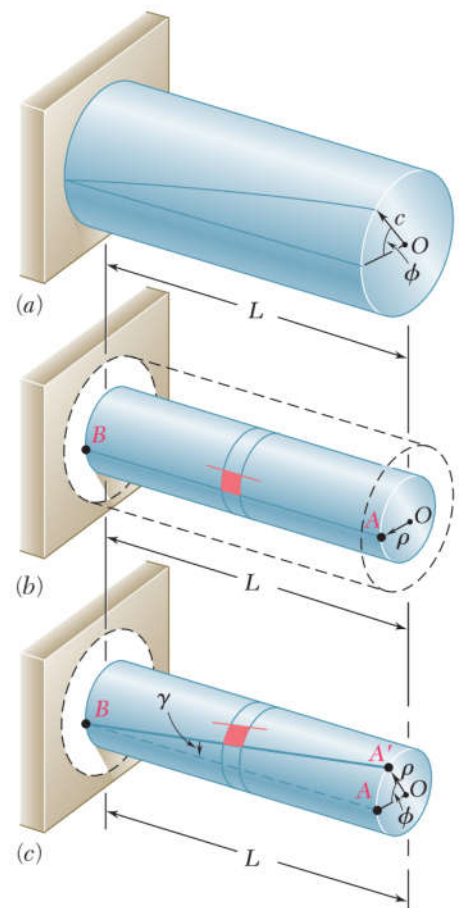


$$T = \int \rho \tau dA = \frac{\tau_{\max}}{c} \int \rho^2 dA$$

$$T = \frac{\tau_{\max} J}{c}$$

$$\tau_{\max} = \frac{Tc}{J}$$

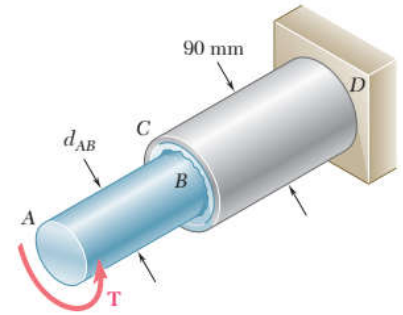
$$\tau = \frac{T\rho}{J}$$





مثال : لنگر پیچشی مجاز

The solid rod AB has a diameter $d_{AB} = 60$ mm. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of steel for which the allowable shearing stress is 75 MPa, determine the largest torque T that can be applied at A .



$$\tau_{all} = 75 \times 10^6 \text{ Pa} \quad T_{all} = \frac{J \tau_{all}}{C}$$

$$\text{Rod } AB: C = \frac{1}{2} d = 0.030 \text{ m}, \quad J = \frac{\pi}{2} C^4$$

$$T_{all} = \frac{\pi}{2} C^3 \tau_{all} = \frac{\pi}{2} (0.030)^3 (75 \times 10^6) \\ = 3.181 \times 10^3 \text{ N}\cdot\text{m}$$

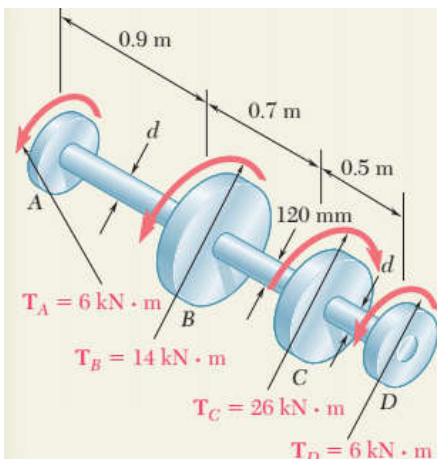
$$\text{Pipe } CD: C_2 = \frac{1}{2} d_2 = 0.045 \text{ m} \quad C_1 = C_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

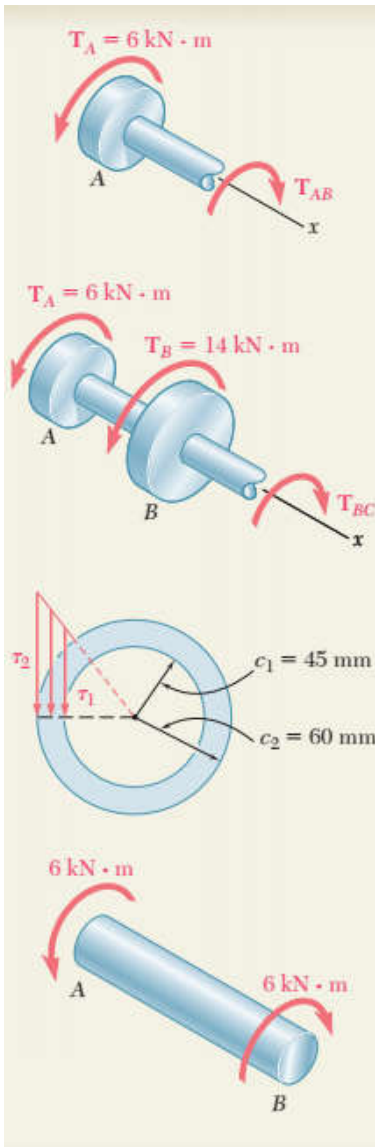
$$T_{all} = \frac{(2.8073 \times 10^{-6})(75 \times 10^6)}{0.045} = 4.679 \times 10^3 \text{ N}\cdot\text{m}$$

Allowable torque is the smaller value $(3.18 \times 10^3 \text{ N}\cdot\text{m})$ $3.18 \text{ kN}\cdot\text{m}$ ◀

مثال : تنش برشی ناشی از پیچش



Shaft BC is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts AB and CD are solid and of diameter d . For the loading shown, determine (a) the maximum and minimum shearing stress in shaft BC , (b) the required diameter d of shafts AB and CD if the allowable shearing stress in these shafts is 65 MPa.

**SOLUTION**

Equations of Statics. Denoting by T_{AB} the torque in shaft AB, we pass a section through shaft AB and, for the free body shown, we write

$$\Sigma M_x = 0: \quad (6 \text{ kN} \cdot \text{m}) - T_{AB} = 0 \quad T_{AB} = 6 \text{ kN} \cdot \text{m}$$

We now pass a section through shaft BC and, for the free body shown, we have

$$\Sigma M_x = 0: \quad (6 \text{ kN} \cdot \text{m}) + (14 \text{ kN} \cdot \text{m}) - T_{BC} = 0 \quad T_{BC} = 20 \text{ kN} \cdot \text{m}$$

a. Shaft BC. For this hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.060)^4 - (0.045)^4] = 13.92 \times 10^{-6} \text{ m}^4$$

Maximum Shearing Stress. On the outer surface, we have

$$\tau_{\max} = \tau_2 = \frac{T_{BC} c_2}{J} = \frac{(20 \text{ kN} \cdot \text{m})(0.060 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4} \quad \tau_{\max} = 86.2 \text{ MPa} \quad \blacktriangleleft$$

Minimum Shearing Stress. We write that the stresses are proportional to the distance from the axis of the shaft.

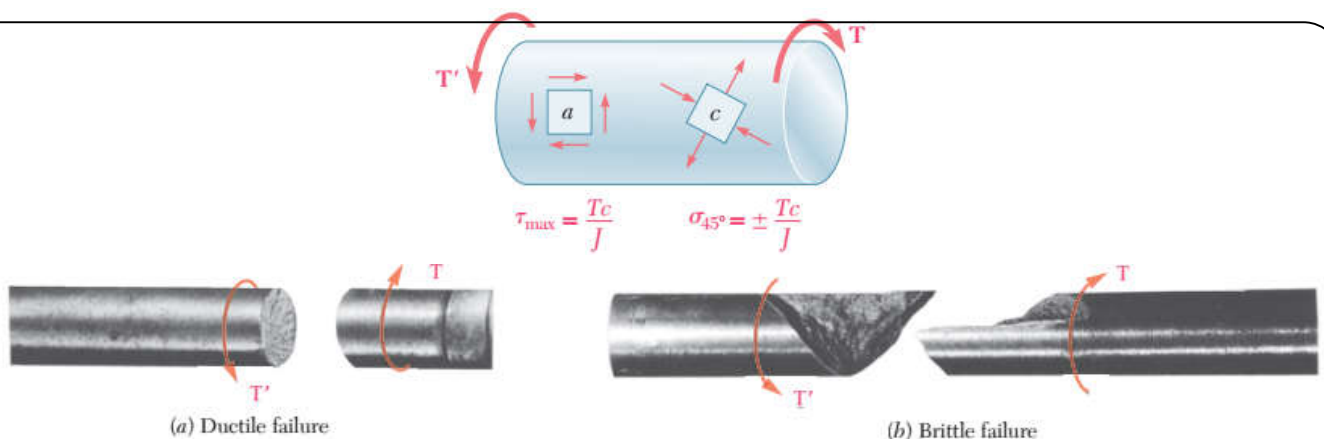
$$\frac{\tau_{\min}}{\tau_{\max}} = \frac{c_1}{c_2} \quad \frac{\tau_{\min}}{86.2 \text{ MPa}} = \frac{45 \text{ mm}}{60 \text{ mm}} \quad \tau_{\min} = 64.7 \text{ MPa} \quad \blacktriangleleft$$

b. Shafts AB and CD. We note that in both of these shafts the magnitude of the torque is $T = 6 \text{ kN} \cdot \text{m}$ and $\tau_{\text{all}} = 65 \text{ MPa}$. Denoting by c the radius of the shafts, we write

$$\tau = \frac{Tc}{J} \quad 65 \text{ MPa} = \frac{(6 \text{ kN} \cdot \text{m})c}{\frac{\pi}{2}c^4}$$

$$c^3 = 58.8 \times 10^{-6} \text{ m}^3 \quad c = 38.9 \times 10^{-3} \text{ m}$$

$$d = 2c = 2(38.9 \text{ mm}) \quad d = 77.8 \text{ mm} \quad \blacktriangleleft$$

۳-۵- مودهای شکست پیچی

شکست در برش در سطح مقطع قائم - مصالح نرم مانند فولاد

شکست در کشش ناشی از برش در سطح مقطع ۴۵ درجه - مصالح ترد مثل گچ یا بتن

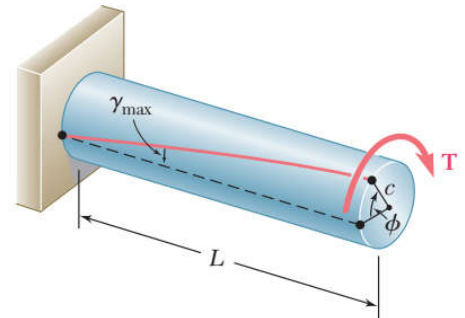


۴-۵- زاویه پیچش در شفت با مقطع دایره

$$\gamma_{\max} = \frac{c\phi}{L}$$

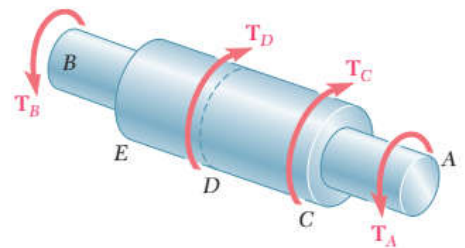
$$\gamma_{\max} = \frac{\tau_{\max}}{G} = \frac{Tc}{JG}$$

$$\phi = \frac{TL}{JG}$$



در مواردی که چند شفت با مشخصات مختلف و یا تحت کوپل های خارجی مختلف قرار بگیرند، باید به قسمت های مجزا تقسیم شده و با رابطه زیر زاویه های پیچش با هم جمع شوند.

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$



دقت شود که Ti لنگر پیچشی داخلی می باشد نه خارجی.

مثال : زاویه پیچش

What torque should be applied to the end of the shaft of Example 3.01 to produce a twist of 2° ? Use the value $G = 77 \text{ GPa}$ for the modulus of rigidity of steel.

Solving Eq. (3.16) for T , we write

$$T = \frac{JG}{L}\phi$$

Substituting the given values

$$G = 77 \times 10^9 \text{ Pa} \quad L = 1.5 \text{ m}$$

$$\phi = 2^\circ \left(\frac{2\pi \text{ rad}}{360^\circ} \right) = 34.9 \times 10^{-3} \text{ rad}$$

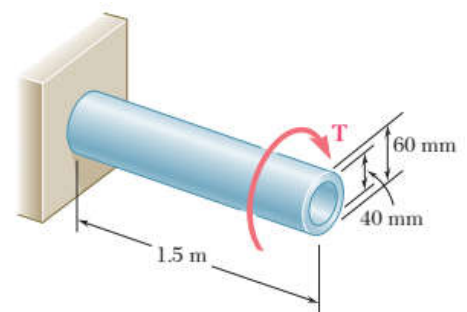
and recalling from Example 3.01 that, for the given cross section,

$$J = 1.021 \times 10^{-6} \text{ m}^4$$

we have

$$T = \frac{JG}{L}\phi = \frac{(1.021 \times 10^{-6} \text{ m}^4)(77 \times 10^9 \text{ Pa})}{1.5 \text{ m}} (34.9 \times 10^{-3} \text{ rad})$$

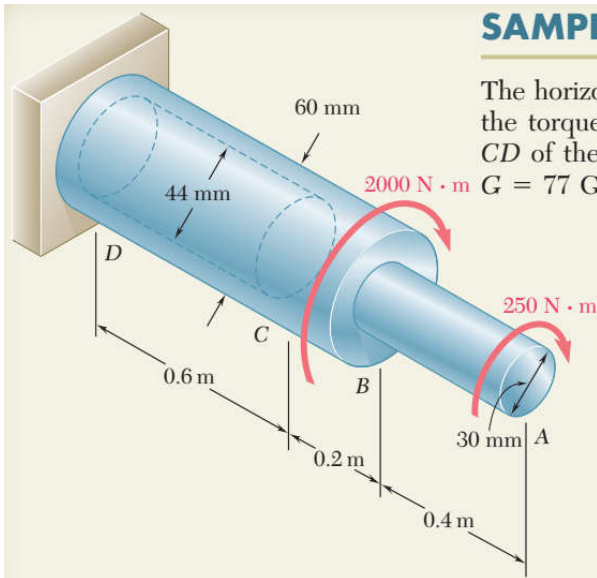
$$T = 1.829 \times 10^3 \text{ N} \cdot \text{m} = 1.829 \text{ kN} \cdot \text{m}$$





مثال : زاویه پیچش

SAMPLE PROBLEM 3.3



The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77 \text{ GPa}$, determine the angle of twist at end A .

SOLUTION

Since the shaft consists of three portions AB , BC , and CD , each of uniform cross section and each with a constant internal torque, Eq. (3.17) may be used.

Statics. Passing a section through the shaft between A and B and using the free body shown, we find

$$\sum M_x = 0: (250 \text{ N} \cdot \text{m}) - T_{AB} = 0 \quad T_{AB} = 250 \text{ N} \cdot \text{m}$$

Passing now a section between B and C , we have

$$\sum M_x = 0: (250 \text{ N} \cdot \text{m}) + (2000 \text{ N} \cdot \text{m}) - T_{BC} = 0 \quad T_{BC} = 2250 \text{ N} \cdot \text{m}$$

Since no torque is applied at C ,

$$T_{CD} = T_{BC} = 2250 \text{ N} \cdot \text{m}$$

Polar Moments of Inertia

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.015 \text{ m})^4 = 0.0795 \times 10^{-6} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.030 \text{ m})^4 = 1.272 \times 10^{-6} \text{ m}^4$$

$$J_{CD} = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} [(0.030 \text{ m})^4 - (0.022 \text{ m})^4] = 0.904 \times 10^{-6} \text{ m}^4$$

Angle of Twist. Using Eq. (3.17) and recalling that $G = 77 \text{ GPa}$ for the entire shaft, we have

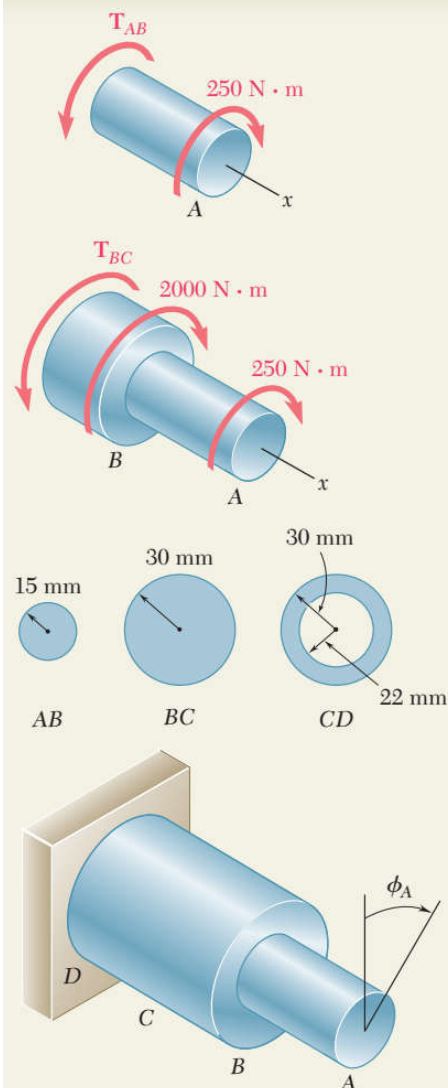
$$\phi_A = \sum_i \frac{T_i L_i}{J_i G} = \frac{1}{G} \left(\frac{T_{AB} L_{AB}}{J_{AB}} + \frac{T_{BC} L_{BC}}{J_{BC}} + \frac{T_{CD} L_{CD}}{J_{CD}} \right)$$

$$\phi_A = \frac{1}{77 \text{ GPa}} \left[\frac{(250 \text{ N} \cdot \text{m})(0.4 \text{ m})}{0.0795 \times 10^{-6} \text{ m}^4} + \frac{(2250)(0.2)}{1.272 \times 10^{-6}} + \frac{(2250)(0.6)}{0.904 \times 10^{-6}} \right]$$

$$= 0.01634 + 0.00459 + 0.01939 = 0.0403 \text{ rad}$$

$$\phi_A = (0.0403 \text{ rad}) \frac{360^\circ}{2\pi \text{ rad}}$$

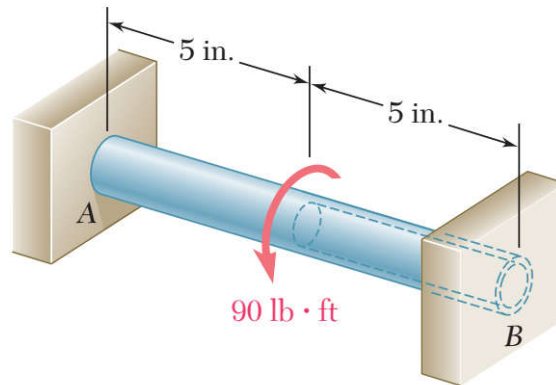
$$\phi_A = 2.31^\circ \quad \blacktriangleleft$$





۵-۵- تحلیل اعضای پیچشی نامعین

A circular shaft AB consists of a 10-in.-long, $\frac{7}{8}$ -in.-diameter steel cylinder, in which a 5-in.-long, $\frac{5}{8}$ -in.-diameter cavity has been drilled from end B . The shaft is attached to fixed supports at both ends, and a $90 \text{ lb} \cdot \text{ft}$ torque is applied at its midsection (Fig. 3.25). Determine the torque exerted on the shaft by each of the supports.



در محلول T_A و T_B مجهول است. معادله ۱ و معادله ۲ را داریم

$$T_A + T_B = 90 \quad \text{معادله ۱}$$

$$\phi_B = 0 \quad \text{معادله ۲}$$

$$\textcircled{2}: \phi_B = \phi_{AC} + \phi_{CB} = \frac{T_{AC} L_{AC}}{G J_{AC}} + \frac{T_{CB} L_{CB}}{G J_{CB}} = 0 \quad \textcircled{3}$$

$$T_{AC} = T_1 = T_A$$

$$T_{CB} = T_A - 90$$

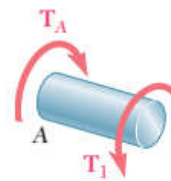
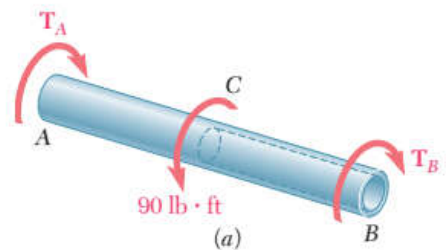
$$L_{AC} = L_{CB} = 5 \text{ in.}$$

$$J_1 = \frac{1}{2} \pi \left(\frac{7}{16} \right)^4 = 57.6 \times 10^{-3} \text{ in}^4$$

$$J_2 = \frac{1}{2} \pi \left[\left(\frac{7}{16} \right)^4 - \left(\frac{5}{16} \right)^4 \right] = 42.6 \times 10^{-3} \text{ in}^4$$

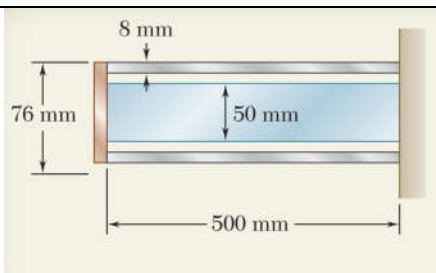
$$\frac{T_A \times 5}{57.6 \times 10^{-3}} + \frac{(T_A - 90) \times 5}{42.6 \times 10^{-3}} = 0 \rightarrow$$

$$T_A = 51.7 \text{ lb.ft} \quad \textcircled{D}: T_B = 90 - 51.7 = 38.3$$



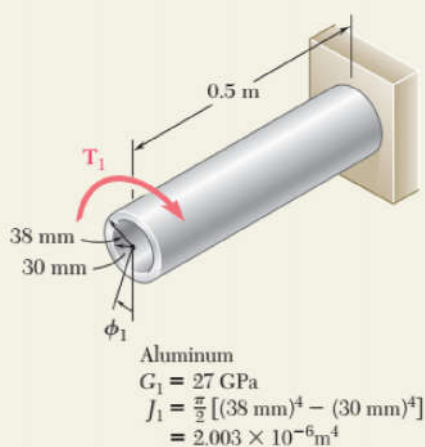
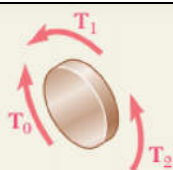


مثال : تحلیل اعضای نامعین پیچشی



شفت مرکب متجانس، از یک میله فولادی دایره‌ای و آلومینیومی مطابق هندسه نشان داده شده تشکیل شده است. انتهای دو عضو در سطح یکدیگر می‌چسبند. اگر کوپل پیچشی $T_o = 7 \text{ kN.m}$ به دایره وارد شود، تنش برشی در شفت ایجاد شده در میله فولادی را بدست آورید.

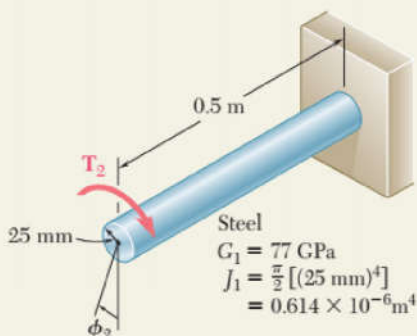
$$G_{\text{steel}} = 77 \text{ GPa}, \quad G_{\text{Alm.}} = 27 \text{ GPa}$$



Aluminum

$$G_1 = 27 \text{ GPa}$$

$$J_1 = \frac{\pi}{2} [(38 \text{ mm})^4 - (30 \text{ mm})^4] = 2.003 \times 10^{-6} \text{ m}^4$$



Steel

$$G_2 = 77 \text{ GPa}$$

$$J_2 = \frac{\pi}{2} [(25 \text{ mm})^4] = 0.614 \times 10^{-6} \text{ m}^4$$

کوپل پیچشی ای برنده در میله T_2 ، کوپل پیچشی ایجاد شده در دایره T_1

$$T_o = T_1 + T_2 \quad (1) \quad \text{معادله استاتیکی (تیر)}$$

$$\phi_1 = \phi_2 \quad (2) \quad \text{معادله گسری تغییر شکل}$$

$$(2) \rightarrow \frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2} \rightarrow \frac{T_1}{T_2} = \left(\frac{L_2}{L_1} \right) \left(\frac{G_1 J_1}{G_2 J_2} \right)$$

$$\rightarrow \frac{T_1}{T_2} = 1 \times \frac{27000 \left(\frac{\text{N}}{\text{mm}^2} \right) \times 2.003 \times 10^6 (\text{mm}^4)}{77000 \left(\frac{\text{N}}{\text{mm}^2} \right) \times 0.614 \times 10^6 (\text{mm}^4)}$$

$$\rightarrow T_2 = 0.874 T_1 \quad (2')$$

$$(1), (2') \rightarrow (1 + 0.874) T_1 = T_o = 7 \text{ kN.m}$$

$$T_1 = \frac{7}{1.874} = 3.735 \text{ kN.m}$$

$$T_2 = 3.265 \text{ kN.m}$$

$$\tau_1 = \frac{T_1 C}{J_1} = \frac{3.735 \times 10^6 (\text{N.mm}) \times 38 (\text{mm})}{2.003 \times 10^6 (\text{mm}^4)}$$

$$\tau_1 = 70.85 \text{ N/mm}^2$$

$$\tau_2 = \frac{T_2 C}{J_2} = \frac{3.265 \times 10^6 (\text{N.mm}) \times 25 (\text{mm})}{0.614 \times 10^6 (\text{mm}^4)}$$

$$\tau_2 = 133 \text{ N/mm}^2$$



۵-۶- پیچش در مقاطع مستطیلی توپر

توزیع تنش برشی ناشی از پیچش در مقاطع توپر مستطیلی بسیار پیچیده است. تنش ماکزیمم در وسط ضلع بزرگتر مقطع رخ می دهد. تنش در گوشه ها صفر است.

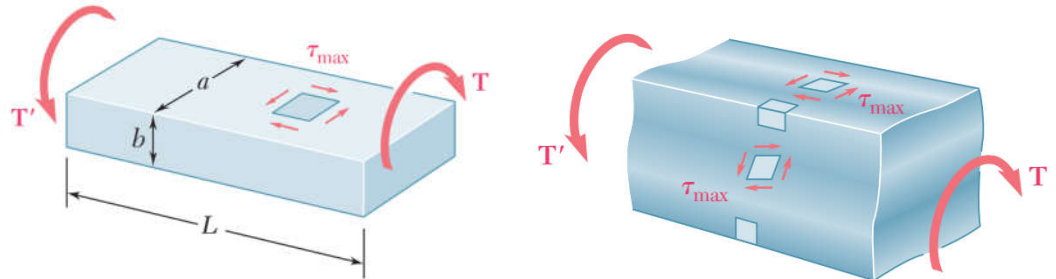


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

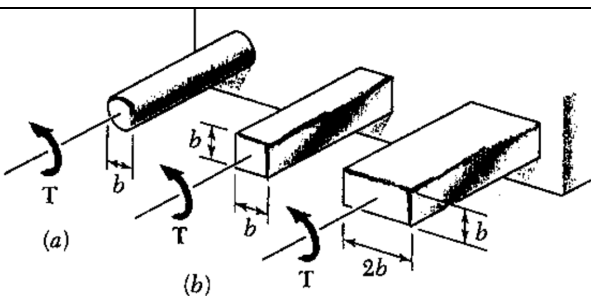
a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

$$\tau_{\max} = \frac{T}{c_1 ab^2}$$

$$\phi = \frac{TL}{c_2 ab^3 G}$$

$$c_1 = c_2 = \frac{1}{3}(1 - 0.630b/a) \quad (\text{for } a/b \geq 5 \text{ only})$$

مثال - پیچش در مقاطع مستطیلی توپر



گفتیم $T = 300 \text{ N.m}$ به مقاطع زیر وارد می شود.
مقدار یابا مقعر b به گونه ای مشخص کنید که تنش برشی حداکثر
ای باشد در مقطع از 60 MPa تجاوز نکند.



④ مقطع دایره‌ای: $\tau = \frac{Tc}{J} = \frac{3 \times 10^5 (N \cdot mm) \times \frac{1}{2}b}{\frac{1}{2} \pi \left(\frac{b}{2}\right)^4} = 60$

$\rightarrow b = 29.4 \text{ mm}$

⑤ مقطع مربع: $a=b \rightarrow \frac{a}{b}=1 \rightarrow C_1=0.208$

$\tau_{max} = \frac{T}{C_1 ab^2} = \frac{T}{C_1 b^3} = \frac{3 \times 10^5 (N \cdot mm)}{0.208 b^3} = 60 \frac{N}{mm^2}$

$\rightarrow b = 28.9 \text{ mm}$

⑥ مقطع مستطیل: $a=2b \rightarrow \frac{a}{b}=2 \rightarrow C_1=0.246$

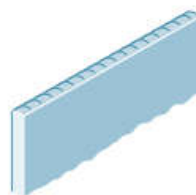
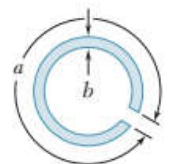
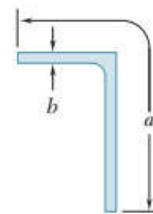
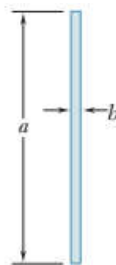
$\tau_{max} = \frac{T}{C_1 ab^2} = \frac{3 \times 10^5 (N \cdot mm)}{0.246 \times 2b^3} = 60 \frac{N}{mm^2}$

$\rightarrow b = 21.7 \text{ mm}$

۵-۷- پیچش در مقاطع نازک باز

در پیچش یک مجو خش نیست $\frac{a}{b}$ بزرگتر از ۱۰ است داریم $C_1 = C_2 = \frac{1}{3}$
 پیچش در مجو خش نازک داریم نیست $\frac{a}{b}$ بزرگتر از ۱۰ است

$\tau_{Max} = \frac{T}{\frac{1}{3}ab^2}$
 $\phi = \frac{T \cdot L}{\frac{1}{3}ab^3 G}$

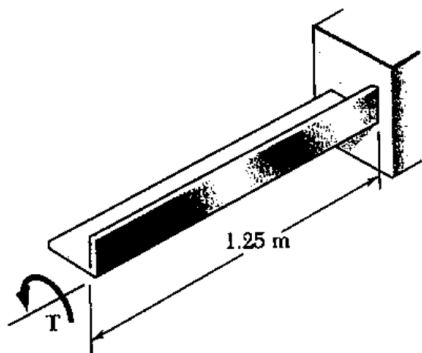




مثال - پیچش در مقاطع نازک باز

یک نبشی به طول 1.25 m مطابق شکل تحت اثر کوپل T قرار دارد.
تنش مجاز برش $\tau_{\text{all}} = 60 \text{ MPa}$ ، $G = 77 \text{ GPa}$ می باشد. محاسبه کنید:
(الف) حداکثر کوپل مجاز

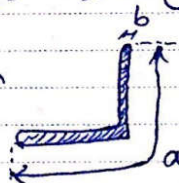
(ب) بیشترین سرنشی تحت کوپل حداکثر مجاز



توضیح: ضخامت ورق نبشی 6.4 mm ، سطح مقطع آن 1252 mm^2 است

نوع مقطع از نظر رفتار پیچشی، مقاطع نازک باز می باشد.

$$b = 6.4 \text{ mm}, \quad a = \frac{A}{b} = \frac{1252}{6.4} = 195.6 \text{ mm}$$



$$a/b = \frac{195.6}{6.4} = 30.56$$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3265$$

$$\tau_{\text{max}} = \frac{T}{C_1 a b^2} \Rightarrow \frac{T}{0.3265 \times 195.6 \times 6.4^2} \leq 60 \frac{\text{N}}{\text{mm}^2}$$

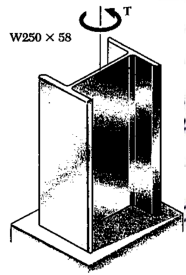
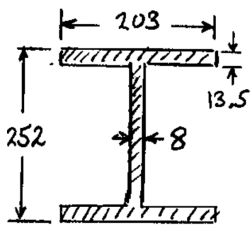
$$T \leq 157 \times 10^6 \text{ N}\cdot\text{mm} \rightarrow \boxed{T_{\text{max}} = 157 \text{ kN}\cdot\text{m}}$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{157 \times 10^6 \text{ (N}\cdot\text{mm)} \times 1250 \text{ (mm)}}{0.3265 \times 195.6 \times 6.4^3 \times 77000}$$

$$\phi = 152.19 \times 10^{-3} \text{ Rad.} \times \left(\frac{360}{2\pi} \right) = 8.72^\circ$$



مثال - پیچش در مقاطع نازک باز - تیر آهن I شکل



یک تیر فولادی H شکل مطابق مشخصات هندسی نشان داده شده ،
 با طول 3 م تحت گسترش T درازگرفته است . اگر $GPa = 77$ ، $\tau_{all} = 35 MPa$
 باشد . مقدار حداکثر گسترش T و زاویه پیچش مشروط با آن را بدست آورید .

از لحاظ پیچش مقطع مورد نظر ، مدار نازک باز مقطع محسوب می شود که از دو قطعه
 بال و یک قطعه جبهه تشکیل شده است . برای حل این مسئله ، ابتدا باید به هم
 هر کدام از قطعات را محاسبه کنیم . برای این کار ، از روش استاندارد مورد
 استفاده (تحلیل انعطاف پذیری) استفاده می کنیم .

$$T = 2T_F + T_W \quad (1) \quad \text{رابطه تادل استاتیکی}$$

$$\phi_F = \phi_W \quad (2) \quad \text{رابطه یکسانی تغییر شکل}$$

$$\phi_F = \frac{T_F L}{C_2 a b^3 G}$$

$$= 0.3194$$

$$a = 203 \text{ mm}, \quad b = 13.5 \text{ mm}, \quad \frac{a}{b} = 15.04, \quad C_1 = C_2 = \frac{1}{3}(1 - 0.63 \frac{b}{a})$$

$$\phi_F = \frac{T_F \times 3000 \text{ (mm)}}{0.3194 \times 203 \times 13.5^3 \times 77000 \text{ (MPa)}} = 2.442 \times 10^{-7} T_F$$

$$\phi_W = \frac{T_W L}{C_2 a b^3 G}$$

$$a = 252 - 2(13.5) = 225, \quad b = 8, \quad \frac{a}{b} = 28.13, \quad C_1 = C_2 = 0.3259$$

$$\phi_W = \frac{T_W \times 3000 \text{ (mm)}}{0.3259 \times 225 \times 8^3 \times 77000 \text{ (MPa)}} = 1.0377 \times 10^{-6} T_W$$

$$(2) \rightarrow \phi_F = \phi_W \rightarrow 2.442 \times 10^{-7} T_F = 1.0377 \times 10^{-6} T_W$$

$$T_F = 4.25 T_W, \quad T_W = \frac{1}{4.25} T_F$$

$$(1) \rightarrow T = 2 \times (4.25) T_W + T_W = 9.5 T_W = \frac{9.5}{4.25} T_F = 2.24 T_F$$

$$\rightarrow T = 9.5 T_W, \quad T = 2.24 T_F$$



$$\tau_F = \frac{T_F}{C_1 a b^2} \leq 35 \text{ MPa} \rightarrow T_F \leq 35 \times 0.3194 \times 203 \times 13.5^2 = 413581 \text{ N.m}$$

$$\tau_w = \frac{T_w}{C_1 a b^2} \leq 35 \text{ MPa} \rightarrow T_w \leq 35 \times 0.3259 \times 225 \times 8^2 = 164253 \text{ N.m}$$

$$T_F \leq 413.581 \text{ N.m} \rightarrow T = 2.24 T_F \leq 2.24 \times 413.581 = 926.4$$

$$T_w \leq 164.25 \text{ N.m} \rightarrow T = 9.5 T_w \leq 9.5 \times 164.253 = 1560$$

$$\rightarrow T \leq 926.4 \text{ N.m}$$

$$\phi = \phi_F = \phi_w = \frac{T_F L}{C_1 a b^3 G} = \frac{413.581 \times 10 \times 3000}{0.3194 \times 203 \times 13.5^3 \times 77000}$$

$$\phi = 0.1 \text{ rad} = 5.73^\circ$$

۵-۸- پیچش در مقاطع نازک بسته

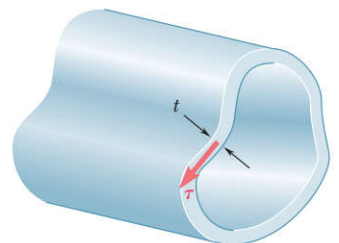
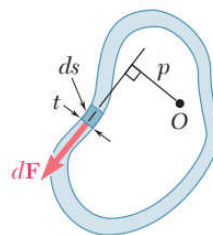
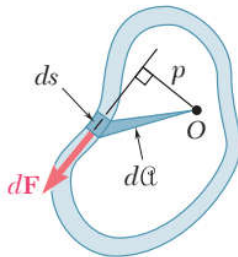
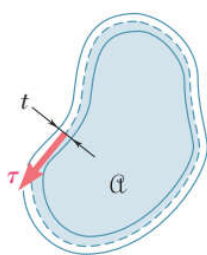
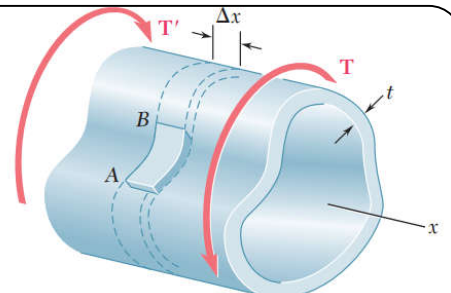
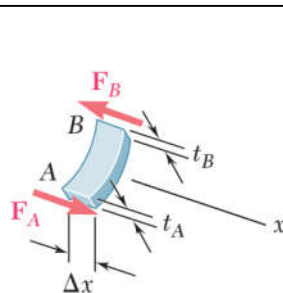
$$F_A - F_B = 0$$

$$F_A = \tau_A (t_A \Delta x)$$

$$\tau_A (t_A \Delta x) - \tau_B (t_B \Delta x) = 0$$

$$\tau_A t_A = \tau_B t_B$$

$$q = \tau t = \text{constant}$$



$$dF = \tau dA = \tau (t ds) = (\tau t) ds = q ds$$

$$dM_O = p dF = p (q ds) = q (p ds)$$

$$dM_O = q (2d\alpha)$$

$$T = \oint dM_O = \oint q (2d\alpha)$$

$$T = 2q\alpha$$

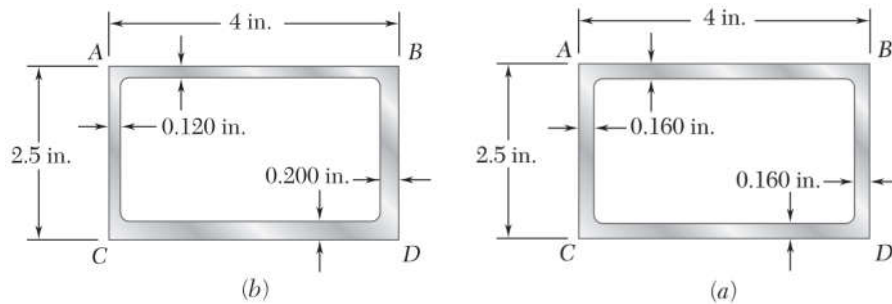
$$\tau = \frac{T}{2t\alpha}$$

$$\phi = \frac{TL}{4\alpha^2 G} \oint \frac{ds}{t}$$



مثال - پیچش در مقاطع جدار نازک بسته

Structural aluminum tubing of 2.5×4 -in. rectangular cross section was fabricated by extrusion. Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of $24 \text{ kip} \cdot \text{in.}$, assuming (a) a uniform 0.160 -in. wall thickness (Fig. 3.55a), (b) that, as a result of defective fabrication, walls AB and AC are 0.120 -in. thick, and walls BD and CD are 0.200 -in. thick (Fig. 3.55b).



(a) Tubing of Uniform Wall Thickness. The area bounded by the center line (Fig. 3.56) is

$$\alpha = (3.84 \text{ in.})(2.34 \text{ in.}) = 8.986 \text{ in}^2$$

Since the thickness of each of the four walls is $t = 0.160 \text{ in.}$, we find from Eq. (3.53) that the shearing stress in each wall is

$$\tau = \frac{T}{2t\alpha} = \frac{24 \text{ kip} \cdot \text{in.}}{2(0.160 \text{ in.})(8.986 \text{ in}^2)} = 8.35 \text{ ksi}$$

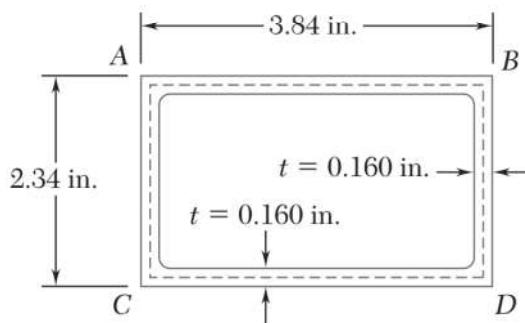


Fig. 3.56

(b) Tubing with Variable Wall Thickness. Observing that the area α bounded by the center line is the same as in part a, and substituting successively $t = 0.120 \text{ in.}$ and $t = 0.200 \text{ in.}$ into Eq. (3.53), we have

$$\tau_{AB} = \tau_{AC} = \frac{24 \text{ kip} \cdot \text{in.}}{2(0.120 \text{ in.})(8.986 \text{ in}^2)} = 11.13 \text{ ksi}$$

and

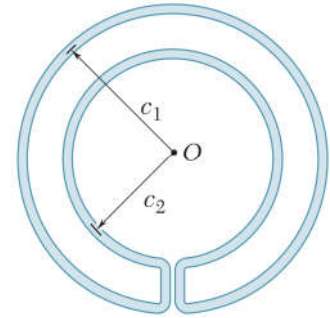
$$\tau_{BD} = \tau_{CD} = \frac{24 \text{ kip} \cdot \text{in.}}{2(0.200 \text{ in.})(8.986 \text{ in}^2)} = 6.68 \text{ ksi}$$

We note that the stress in a given wall depends only upon its thickness.



مثال - پیچش در مقاطع جدار نازک بسته

A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (a) the maximum shearing stress in the tube, (b) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.



Area bounded by centerline

$$A = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2 \\ = 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

$$(a) \tau = \frac{T}{2tA} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} = 12.73 \text{ MPa} \quad \blacktriangleleft$$

$$(b) T_i = (2\pi c_i t \tau_{c_i}) = 2\pi c_i^2 t \tau$$

$$= 2\pi (0.150)^2 (0.003) (12.73 \times 10^6) = 5.46 \times 10^3 \text{ N}\cdot\text{m} \\ = 5.46 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$